# Influence of an Externally Modulated Photonic Link on a Microwave Communications System

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We analyze the influence of an externally modulated photonic link on the performance of a microwave communications system. From the analysis, we deduce limitations on the photocurrent, magnitude of the relaxation oscillation noise of the laser, third-order intercept point of the preamplifier, and other parameters in order for the photonic link to function according to the system specifications. Based on this, we outline a procedure for designing a photonic link that can be integrated in a system with minimal performance degradation.

#### I. Introduction

Photonic technology has become increasingly important in analog communications systems. For systems with high frequency and high dynamic range, externally modulated photonic links generally have better performance compared to directly modulated links [1,2]. The performance of such links has been analyzed by many authors [3-6]; however, in these analyses the links were assumed to be isolated from the microwave system and, therefore, their effect on the system was not adequately apparent. In addition, many parameters in these analyses were given for component engineers and, thus, are difficult to use for a system designer, who may not be familiar with the photonic technology. Finally, none of the analyses considered the influence of the laser's relaxation oscillation noise amplitude on the microwave system, which, as will be discussed later, may be critical in many applications.

We present here an analysis that emphasizes the integration of the link in an analog system. We pay special attention to the laser's relaxation oscillation noise and determine quantitatively its effect on the system. With parameters and equations intentionally written in system engineering terms, we hope that the results can be readily used by microwave engineers in their system designs.

An analog communications system can be considered as many subsystems that are cascaded together. Each subsystem i has a characteristic gain  $G_i$ , noise factor  $F_i$ , 1-dB compression  $P_i^{1dB}$ , third-order intercept point  $IP_i$ , and bandwidth  $\Delta f_i$ . Grouping the subsystems is somewhat arbitrary; for convenience, we group the system under consideration into three subsystems, as shown in Fig. 1. All the components before the optical link are included in subsystem 1, and all the components after the optical link are included in subsystem 2. For example, in an antenna remote system where the optical link is inserted between the low-noise amplifier (LNA) of the antenna and the downconverter, subsystem 1 is the LNA and subsystem 3 includes the downconverter

and all the components following the downconverter. With such a grouping, the effect of the optical link on the system's performance can easily be evaluated.

In this article, we first determine the gain, noise figure, 1-dB compression, and intercept point of an isolated optical link (or subsystem 2). We then determine quantitatively the changes in noise figure, dynamic range, and gain profile of the system caused by the insertion of the optical link. From the analysis, we deduce limitations on the magnitude of the relaxation oscillation peak, photocurrent, and other parameters of the optical link in order for the link to function according to the system's specifications. Finally, based on the analysis, we outline a procedure for designing a photonic link that can be integrated in a system with minimal performance degradation.

#### II. Parameters of an Isolated Optical Link

#### A. Gain of the Photonic Link

The photonic link consists of an electro-optic (E/O) modulator to convert the RF signal into an optical signal, a length of optical fiber to transmit the optical signal, and an optical receiver to convert the optical signal back to RF. Because two signal conversion processes are involved, the signal loss is generally high. The attenuation of the optical signal in the fiber also produces additional RF loss. The total RF gain (or loss) of the photonic link using the Mach-Zehnder modulator [5] is (see Appendix A)

$$G_{op} = \pi^2 \frac{I_{ph}^2 R_L}{V_{\pi}^2 / R_m} \tag{1}$$

where  $V_{\pi}$  is the half-wave voltage of the modulator,  $R_{L}$  is the load impedance of the receiver, and  $I_{ph}$  is the average photocurrent in the load resistor of the receiver. In Eq. (1), the numerator is the electrical power generated by the photocurrent in the receiver, and the denominator is the input electrical power to the modulator corresponding to an applied voltage of  $V_{\pi}$ . The photocurrent in the receiver is related to the received optical power  $W_{o}$  by  $I_{ph} = \eta W_{o}$ , where  $\eta$  is the responsivity of the receiver.

# B. The 1-dB Compression and Third-Order Intercept Point of the Optical Link

Referring to Appendix A, one can see that the input 1-dB compression  $P_m^{\text{1dB}}$  and third-order intercept point  $IP_m$  of a Mach-Zehnder modulator are

$$IP_m = 10P_m^{\text{1dB}} = \frac{4}{\pi^2} \frac{V_\pi^2}{R_m} = \frac{4I_{ph}^2 R_L}{G_{op}}$$
 (2)

They can be converted to the results obtained by Kolner and Dolfi in [7]. From Eq. (2), one can see that for a Mach–Zehnder modulator the third-order intercept point is always 10 times (or 10-dB) higher than the 1-dB compression. For a modulator with  $V_{\pi}$  of 8 V and  $R_{m}$  of 50  $\Omega$ , the 1-dB compression is 17 dBm, and the third-order intercept point is 27 dBm.

The output intercept  $IP_{op}$  is simply the product of the input intercept and the gain of the link. From Eq. (2), we can readily obtain

$$IP_{op} = 10P_{op}^{1dB} = 4I_{ph}^2 R_L$$
 (3)

where  $P_{op}^{1dB}$  is the output 1-dB compression of the link. It is important to notice that the output 1-dB compression and the third-order intercept are independent of the characteristics of the modulator and are proportional to the photo-electric power in the receiver.

#### C. Preamplifier Requirements

Typically, the loss of the basic optical link is -20 to -60 dB. To compensate for the signal loss, an amplifier with a gain of

$$G_{pr} = \frac{1}{G_{op}} \tag{4}$$

may be placed either before the modulator or after the receiver. However, a preamplifier is preferred because it also serves to reduce the noise figure of the optical link.

In order for the preamplifier not to limit the dynamic range of the photonic link, its output intercept  $IP_{pr}$  and 1-dB compression  $P_{pr}^{1dB}$  must be much larger than the input intercept  $IP_m$  and 1-dB compression  $P_m^{1dB}$  of the modulator, respectively. Using Eq. (2), we therefore obtain

$$IP_{pr} >> \frac{4}{\pi^2} \frac{V_{\pi}^2}{R_m}$$
 (5)

$$P_{pr}^{1dB} >> \frac{2}{5\pi^2} \frac{V_{\pi}^2}{R_m}$$
 (6)

### D. Third-Order Intercept Point of the Loss-Compensated Optical Link

Using the cascading formula given by Norton [8], we obtain the output third-order intercept  $IP_2$  of the loss-compensated photonic link (subsystem 2):

$$\frac{1}{IP_2} = \frac{1}{G_{op}IP_{pr}} + \frac{1}{IP_{op}} \tag{7}$$

Because the loss-compensated optical link has a gain of unity,  $G_2 \equiv G_{pr}G_{op} = 1$ ,  $IP_2$  is also the input intercept of the link. When Eq. (5) is satisfied,  $IP_2 = IP_{op}$  and both the input and output intercept points of the loss-compensated link are equal to the output intercept point of the isolated optical link. Because  $IP_{op} = 4I_{ph}^2R_L$ , again the intercept of the loss-compensated link is independent of the characteristics of the modulator and depends on only the photo-electric power in the receiver.

Similarly, the input and output 1-dB compression  $P_2^{1dB}$  of the loss-compensated link is

$$\frac{1}{P_2^{1dB}} = \frac{1}{G_{op}P_{pr}^{1dB}} + \frac{1}{P_{op}^{1dB}}$$
 (8)

### E. Noise of the Optical Link

The total noise density (per hertz) at the output of the fiber optical link is

$$\mathcal{P}_{op} = G_{op}\mathcal{P}_{m} + \mathcal{P}_{thermal} + \mathcal{P}_{shot} + \mathcal{P}_{RIN}$$
 (9)

where  $\mathcal{P}_m$  is the input noise density to the modulator,  $\mathcal{P}_{thermal}$  is the thermal noise density generated in the receiver,  $\mathcal{P}_{shot}$  is the shot noise density, and  $\mathcal{P}_{RIN}$  is the relative intensity noise density generated by the laser around the modulation frequency  $f_m$ . In Eq. (9), we neglected the dark current noise generated by the photodetector because the anticipated photocurrent (approximately 1 mA) is much larger than the dark current (approximately 1 nA). The thermal noise and the shot noise are white noise processes, and their expressions are well known, as follows:

$$\mathcal{P}_{thermal} = kT_{op} \tag{10}$$

$$\mathcal{P}_{shot} = 2eI_{ph}R_L \tag{11}$$

where  $T_{op}$  is the ambient temperature of the optical link, k is Boltzmann's constant, and e is the charge of the electron.

The relative intensity noise (RIN) of a YAG laser is frequency dependent and has a relaxation oscillation peak around a few hundred kHz [9]. However, this low frequency noise peak can be multiplied up to the modulation frequency by the modulator and contributes to the total relative intensity noise around  $f_m$ , as shown in Fig. 2(a). To find the relationship between the amplitudes of the multiplied RIN peak and the modulation, we performed a simple experiment, as shown in Fig. 2(b). We first added a small single-tone modulation to the laser light with a modulator (modulator 1) at a frequency (250 kHz) close to that of the relaxation oscillation peak (187 kHz). A second modulator (modulator 2) was then used to impose a strong modulation at a higher frequency. As one can see in Fig. 2(c), both the relaxation oscillation peak and the single tone were multiplied up and their relative amplitudes remained unchanged. This result indicates that the amplitude of the multiplied RIN peak can be calculated by treating the relaxation oscillation peak of the RIN as a single-tone modulation, as is done in Appendix B.

Consequently, the total RIN,  $\mathcal{P}_{RIN}$ , at the frequency of interest, f, is the sum of the baseband relaxation oscillation,  $\mathcal{P}_{oRIN}$ , at f and the multiplied relaxation oscillation peak,  $\mathcal{P}_{mRIN}$ :

$$\mathcal{P}_{RIN} = \mathcal{P}_{oRIN} + \mathcal{P}_{mRIN} \tag{12}$$

where

$$\mathcal{P}_{oRIN} = I_{ph}^2 R_L RIN \left( f_m \right) \tag{13}$$

$$\mathcal{P}_{mRIN} = \frac{I}{4} G_{op} P_m RIN \left( f - f_m \right) \tag{14}$$

In Eqs. (13) and (14), RIN(f) is the laser RIN fluctuation at f and  $RIN(f-f_m)$  is the RIN fluctuation around relaxation oscillation frequency  $f_{RLX}$ . They have a unit of 1/Hz. Note that  $RIN(f-f_m)$  is a strong function of frequency around its peaks at  $f-f_m=\pm f_{RLX}$ , as shown in Fig. 2(a). From Eq. (14), one can see that the larger the driving signal, the more the low-frequency relaxation oscillation noise contributes to the noise at the modulation frequency,  $f_m$ .

From Eqs. (10), (11), and (13), one can see that the thermal noise is independent of photocurrent  $I_{ph}$ , the shot noise is proportional to  $I_{ph}$ , and the RIN is proportional to  $I_{ph}$  squared. Comparing  $\mathcal{P}_{thermal}$ ,  $\mathcal{P}_{shot}$ , and  $\mathcal{P}_{oRIN}$ , one can see that at low photocurrent ( $I_{ph} \leq 0.25 \text{ mA}$ ), the

thermal noise  $\mathcal{P}_{thermal}$  dominates. At moderate photocurrent (0.25 mA  $\leq I_{ph} \leq 10$  mA), the shot noise  $\mathcal{P}_{shot}$  dominates, and at high photocurrent ( $I_{ph} \geq 10$  mA), the RIN  $\mathcal{P}_{oRIN}$  dominates. In the calculation above, a relative intensity fluctuation RIN(f) of -165 dB/Hz and  $R_L = 50~\Omega$  are assumed. Most laser manufacturers use this number to conservatively specify the RIN for diode-pumped YAG lasers at above 10 MHz; no definitive measurement has been performed so far to accurately determine it. It is generally believed that the actual value of the RIN for diode-pumped YAG lasers can be much smaller than -165 dB/Hz.

#### F. RIN Noise Peak

Since the multiplied relaxation-oscillation noise peaks are a few hundred kHz away from the modulation frequency, with their amplitudes increasing as the driving signal increases, it is very easy to mistake them for the signal [see Fig. 2(a)]. Thus, for a practical system, these peaks have to be suppressed below the system noise floor. The RIN fluctuation at relaxation oscillation frequency  $RIN(f_{RLX})$  of a diode-pumped YAG laser without a noise reduction circuit is typically as high as -100 dB/Hz. Assuming that  $V_{\pi}$  of the modulator is 8 V,  $R_L$  and  $R_m$  are 50  $\Omega$ ; then, with a moderate driving signal level of 1 mW (corresponding to a modulation depth of 1.54 percent) and a photocurrent of 0.02 mA, the multiplied relaxation oscillation peaks will be above the noise floor, set by the sum of the thermal, the shot, and the original RIN noise terms. The maximum multiplied RIN noise density at the peak,  $f - f_m = \pm f_{RLX}$ , can be obtained by replacing  $P_m$  in Eq. (14) with  $P_m^{\text{max}}$ , the maximum allowed driving power of the system at the modulator:

$$\mathcal{P}_{mRIN}^{\max} = \frac{1}{4} G_{op} P_m^{\max} RIN \left( f_{RLX} \right) \tag{15}$$

If the input noise  $\mathcal{P}_m$  to the optical link is small compared to the other noise terms, and the maximum driving power of the system at the modulator is  $P_m^{\max} = P_m^{\text{1dB}}$ , then in order for this multiplied RIN to be below the noise floor of the link,  $RIN(f_{RLX})$  should satisfy

$$RIN(f_{RLX}) \le \frac{10kT_{op}}{I_{ph}^2R_L} + \frac{20e}{I_{ph}} + 10RIN(f_m)$$
 (16)

However, in an actual system, the input noise  $\mathcal{P}_m$  to the optical link [the first noise term in Eq. (9)] is much larger than the rest of the noise terms and sets the noise

floor of the system. In order for the multiplied RIN to be 10 dB lower than the input noise floor, we must have

$$RIN\left(f_{RLX}\right) \le \frac{0.4}{D_{sus}} \tag{17}$$

where  $D_{sys} = P_m^{\rm max}/\mathcal{P}_m$  is the maximum signal-to-noise ratio of the system, or simply the dynamic range. For example, if the dynamic range of the system is 131 dB-Hz, the RIN peak must be smaller than -135 dB/Hz. Diodepumped YAG lasers with relaxation oscillation peaks at this level may now be obtained commercially. These lasers reduce the RIN peak amplitude by incorporating a feedback loop in the pump diode circuit [9]. For systems requiring higher dynamic range, further noise reduction with an external circuit [10] may be necessary.

# G. Noise Factor of the Loss-Compensated Optical Link

The noise factor of the basic optical link may be expressed as

$$F_{op} = \frac{\mathcal{P}_{op}}{G_{op}\mathcal{P}_m} = 1 + \frac{F_{th} + F_{shot} + F_{oRIN} + F_{mRIN}}{G_{op}} \tag{18}$$

where  $F_{th}$ ,  $F_{shot}$ ,  $F_{oRIN}$ , and  $F_{mRIN}$  are the noise factor contributions from the thermal, shot, RIN, and the multiplied RIN noise, respectively. They are given by the following expressions:

$$F_{th} = \frac{\mathcal{P}_{thermal}}{\mathcal{P}_{m}} = 1 \tag{19a}$$

$$F_{shot} = \frac{\mathcal{P}_{shot}}{\mathcal{P}_m} = \frac{2eI_{ph}R_L}{kT_{op}}$$
 (19b)

$$F_{oRIN} = \frac{\mathcal{P}_{oRIN}}{\mathcal{P}_m} = \frac{I_{ph}^2 R_L}{k T_{op}} RIN(f_m)$$
 (19c)

$$F_{mRIN}(f) = \frac{\mathcal{P}_{mRIN}}{\mathcal{P}_{m}} = \frac{G_{op}}{4kT_{op}} P_{m}RIN(f - f_{m}) \quad (19d)$$

where  $\mathcal{P}_m = kT_{op}$  is used throughout. Because  $G_{op}$  is small (approximately  $10^{-2} - 10^{-6}$ ), the resulting noise factor,  $F_{op}$ , is large; thus, to reduce the noise factor, a preamplifier is required. With the preamplifier gain of  $G_{pr}$  and

noise factor of  $F_{pr}$ , the noise factor of the loss-compensated optical link is

$$F_2(f) = F_{pr} + \frac{F_{op} - 1}{G_{pr}} = F_{2o} + F_{mRIN}(f)$$
 (20)

where

$$F_{2o} = F_{pr} + F_{th} + F_{shot} + F_{oRIN}$$
 (21)

is the frequency-independent part of the noise factor and  $F_{mRIN}(f)$  is the frequency-dependent part.

#### H. Compression Dynamic Range of the Optical Link

The dynamic range of a system is defined as the maximum output signal power divided by the total output noise of the system:  $D_{op} = P_{op}^{1dB}/\mathcal{P}_{op}$ , or

$$\frac{1}{D_{op}} = \frac{1}{D_m} + \frac{1}{D_{thermal}} + \frac{1}{D_{shot}} + \frac{1}{D_{oBIN}} + \frac{1}{D_{mBIN}} \tag{22}$$

where

$$D_m = \frac{P_{op}^{1\text{dB}}}{G_{on}\mathcal{P}_m} = \frac{P_m^{1\text{dB}}}{\mathcal{P}_m}$$
 (23a)

$$D_{th} = \frac{P_{op}^{1dB}}{\mathcal{P}_{thermal}} = \frac{2}{5} \frac{I_{ph}^2 R_L}{kT_{op}}$$
 (23b)

$$D_{shot} = \frac{P_{op}^{1dB}}{P_{shot}} = \frac{I_{ph}}{5e}$$
 (23c)

$$D_{oRIN} = \frac{P_{op}^{1dB}}{\mathcal{P}_{oRIN}} = \frac{2}{5RIN(f_m)}$$
 (23d)

$$D_{mRIN} = \frac{P_{op}^{\text{1dB}}}{P_{mRIN}^{\text{max}}} = \frac{4}{RIN(f_{RLX})}$$
 (23e)

Equations (9) through (14) are used in deriving these equations. To express in dB-Hz, the above equations can be rewritten as

$$D_{th} = 157 + 20 \log I_{ph}(\text{mA}) \tag{24a}$$

$$D_{shot} = 151 + 10 \log I_{ph}(\text{mA})$$
 (24b)

$$D_{oRIN} = -4 - 10 \log RIN(f_m) \qquad (24c)$$

$$D_{mRIN} = 6 - 10 \log RIN(f_{FLX}) \tag{24d}$$

In the above calculations, we assume that  $T_{op} = 290$  K and  $R_L = 50$   $\Omega$ .

#### I. Spur-Free Dynamic Range of an Optical Link

The spur-free dynamic range  $SFD_{op}$  [in units of  $(Hz)^{2/3}$ ] of the optical link is

$$SFD_{op} = \left(\frac{IP_{op}}{\mathcal{P}_{op}}\right)^{2/3} \tag{25}$$

where  $IP_{op}$  is the output third-order intercept of the modulator and is defined in Eq. (3). Since  $IP_{op} = 10P_{op}^{1dB}$ , we have the following simple relation between the compression dynamic range and spur-free dynamic range of an optical link:

$$SFD_{op} = (10D_{op})^{2/3}$$
 in units of  $(Hz)^{2/3}$  (26)

or

$$SFD_{op} = 6.7 + \frac{2}{3}D_{op}$$
 in units of dB-(Hz)<sup>2/3</sup> (27)

# III. Influence of the Optical Link on the System

The ultimate optical link for antenna remoting does not degrade the performance of the existing system and will remain essentially "transparent" to the system. Thus, the insertion of such an optical link in the existing system does not change the system's noise temperature, dynamic range, gain profile, and phase noise. Such an optical link is the basis of our analysis below, where the influence of various parameters is considered in order to determine the effect of each component of the link separately and to specify the required parameters to achieve "transparent" operation upon insertion in the system.

#### A. Influence on the Noise Factor

The noise factor of the system without the optical link is

$$F_{sys} = F_1 + \frac{T_3}{T_1} \frac{F_3 - 1}{G_1} \tag{28}$$

where  $G_1$ ,  $F_1$ , and  $T_1$  are the gain, noise factor, and input noise temperature of subsystem 1, and where  $F_3$  and  $T_3$  are the noise factor and input noise temperature of subsystem 3.

With the optical link inserted, the total noise factor is

$$F'_{sys} = F_1 + \frac{F_2 - 1}{G_1} \frac{T_2}{T_1} + \frac{F_3 - 1}{G_1 G_2} \frac{T_3}{T_1}$$
 (29)

Because  $G_2 = 1$  for a loss-compensated optical link, the total noise factor of the system is, therefore,

$$F'_{sys} = F_{sys} + \Delta F_2 \tag{30}$$

where

$$\Delta F_2 \equiv \Delta F_{2o} + \Delta F_{mRIN} \tag{31}$$

is the total noise factor increase caused by the insertion of the optical link. In Eq. (31),  $\Delta F_{2o} \equiv (T_2/T_1) \times (F_{2o}-1)/G_1$  is the frequency-independent part of the noise factor increase caused by the preamplifier, thermal, shot, and baseband RIN. It can be expressed as

$$\Delta F_{2o} =$$

$$\frac{T_2 F_{pr} + 2e I_{ph} R_L / k + I_{ph}^2 R_L RIN(f_m) / k}{G_1 T_1}$$
 (32)

where  $T_{op} = T_2$  is used. Taking  $R_L = 50 \Omega$ ,  $RIN(f_m) = -165 \text{ dB/Hz}$ , and  $I_{ph}$  in units of mA, Eq. (32) becomes

$$\Delta F_{2o} \approx \frac{T_2 F_{pr} + 1160 I_{ph} + 116 I_{ph}^2}{G_1 T_1}$$
 (33)

On the other hand,  $\Delta F_{mRIN}$  is the frequency-dependent part of the noise factor and is induced by the multiplied RIN noise peak. It is defined as

$$\Delta F_{mRIN} \equiv \frac{1}{4} \frac{P_m/G_1 G_{pr}}{kT_1} RIN \left( f - f_m \right) \tag{34}$$

In Eq. (34),  $P_m/G_1G_{pr}$  is the input signal power of the system, and  $kT_1$  is the input noise density to the system.

The ratio of the two terms is just the input signal-to-noise ratio (SNR) in a 1-Hz bandwidth. Note that  $P_m/G_1G_{pr}$  can be either the signal of interest or an interference signal that falls into the pass band of the signal of interest. When there is no interference signal present, this term is not critical because, when the signal of interest is small, this term is small. Although this term becomes very large at high signal levels, the resulting noise factor increase is not damaging because the corresponding SNR is so high that a small noise increase will have little effect on the performance of the system.

On the other hand, when there is a strong interference signal present,  $P_m/G_1G_{pr}$  will be large, even though the signal of interest is small. Consequently, the SNR of the signal of interest is reduced, and the performance of the system is degraded.

The largest  $\Delta F_{mRIN}$  occurs when  $f - f_m = \pm f_{RLX}$  and when the SNR of the system is a maximum:

$$\Delta F_{mRIN}^{\text{max}} = \frac{1}{4} F_{sys} D_{sys} RIN(f_{RLX})$$
 (35)

Here,  $F_{sys}$  is the noise figure of the system, and  $D_{sys}$  is the dynamic range of the system or the maximum SNR.

#### B. Influence on the Dynamic Range of the System

Without the optical link, the original input third-order intercept point  $IP_{sys}$  of the system is

$$\frac{1}{IP_{SUS}} = \frac{1}{IP_1} + \frac{G_1}{IP_3} \tag{36}$$

where  $IP_3$  is the input intercept point of subsystem 3.

After the insertion of the optical link, the third-order intercept point of the whole system is

$$\frac{1}{IP'_{eve}} = \frac{1}{IP_1} + \frac{G_1}{IP_2} + \frac{G_1G_2}{IP_3} \tag{37}$$

where  $IP_2$  is the input intercept point of the optical link and is given by Eq. (7). For the loss-compensated optical link,  $G_2 \equiv G_{pr}G_{op} = 1$ , Eq. (37) becomes

$$IP'_{sys} = \frac{IP_{sys}}{1 + G_1 IP_{sys}/IP_2} \tag{38}$$

The reduction of the system's intercept point in dB is, therefore,

$$\Delta(IP) = 10 \log \left( 1 + \frac{G_1 I P_{sys}}{I P_2} \right) \tag{39}$$

Similarly, the 1-dB compression,  $P'_{sys}$ , of the system after the insertion of the optical link is

$$P'_{sys} = \frac{IP_{sys}^{1dB}}{1 + G_1 P_{sys}^{1dB} / P_2^{1dB}}$$
(40)

and the reduction of the system's 1-dB compression in dB is

$$\Delta \left(P^{1dB}\right) = 10 \log \left(1 + \frac{G_1 P_{sys}^{1dB}}{P_2^{1dB}}\right) \tag{41}$$

Finally, the spur-free dynamic range and compression dynamic range of the system after the insertion of the optical link can be expressed as

$$SFD'_{sys} = \left(\frac{IP'_{sys}}{F'_{sys}kT_1}\right)^{2/3}$$

$$= \frac{SFD_{sys}}{\left[(1 + \Delta F_2/F_{sys})(1 + G_1IP_{sys}/IP_2)\right]^{2/3}}$$
(42)

$$D'_{sys} = \frac{P'_{sys}}{F'_{sys}kT_1} = \frac{D_{sys}}{(1 + \Delta F_2/F_{sys})(1 + G_1P_{sys}^{1dB}/P_2^{1dB})}$$
(43)

where Eqs. (30), (38), and (40) are used. The degradation of the spur-free dynamic range (in dB-Hz<sup>2/3</sup>) and the compression dynamic range (in dB-Hz) of the system are, therefore,

$$\Delta (SFD_{sys}) \approx \left(\frac{2}{3}\right) \Delta (IP) \approx 6.7 \log \left(1 + \frac{G_1 IP_{sys}}{IP_2}\right)$$
(44)

$$\Delta D_{sys} \approx \Delta \left(P^{\text{1dB}}\right) = 10 \log \left(1 + \frac{G_1 P_{sys}^{\text{1dB}}}{P_2^{\text{1dB}}}\right)$$
 (45)

where  $\Delta F_{op}/F_{sys} \ll 1$  is assumed.

When the preamplifier is properly chosen so that Eq. (5) is satisfied, then from Eqs. (7) and (8) we have  $IP_2 = 4I_{ph}^2R_L$  and  $P_2^{1dB} = 2I_{ph}^2R_L/5$ . Consequently, the dynamic range of the system is solely limited by the photoelectric power. The higher the photoelectric power, the smaller the degradation. On the other hand, if the requirement of Eq. (5) cannot be satisfied, then both the preamplifier and the photoelectric power limit the system's dynamic range.

#### C. Photocurrent Requirement

If the preamplifier is properly chosen so that Eq. (5) is satisfied, then from Eq. (44), in order for the degradation  $\Delta(SFD_{sys})$  of the spur-free dynamic range to be less than 1 dB, the photocurrent must be

$$I_{ph} \ge \sqrt{\frac{5G_1 I P_{sys}}{8R_L}} \tag{46}$$

Similarly, from Eq. (45), in order for the degradation,  $\Delta D_{sys}$ , of the compression dynamic range to be less than 3 dB, the photocurrent must be

$$I_{ph} \ge \sqrt{\frac{5G_1 P_{sys}^{1dB}}{2R_L}} \tag{47}$$

From Eqs. (46) and (47), one can see that the higher the input 1-dB compression of the system, the higher the optical power (photocurrent) of the optical link is required to be to preserve the dynamic range of the system.

# **IV. Summary**

We analyzed the influence of an externally modulated fiber-optic link on a microwave communications system and determined quantitatively the degradation of the noise figure and the dynamic range caused by inserting the link in the system. We found that if the preamplifier is properly chosen, the photo-electric power in the photodetector is the only parameter of the link that affects the dynamic range of the system. The higher the photo-electric power, the less dynamic range degradation of the system. For a system of a given dynamic range, we deduced the minimum photocurrent (optical power) requirement for the system.

We also determined quantitatively the effect of different noise terms of the optical link on the noise figure of the system. We paid special attention to the laser's relaxation oscillation noise and showed how it is multiplied up in frequency by the modulation signal to degrade the signal. The maximum amplitude of the relaxation oscillation peak is inversely proportional to the achievable dynamic range of the system.

Finally, we studied the requirements of the preamplifier required for the optical link. Both the required gain and the third-order intercept of the preamplifier are proportional to  $V_{\pi}^2/R_m$ , a quantity solely determined by the characteristics of the modulator. In practice, modulators with small  $V_{\pi}$  and large  $R_m$  should be used so that the preamplifier can meet the requirements of Eqs. (4) and (5).

When designing a photonic link for analog communications systems, the following procedure is recommended.

First, a laser with low relaxation oscillation noise should be chosen according to Eq. (17). This ensures that the multiplied noise peaks are well below the noise floor of the system. Second, the minimum photocurrent  $I_{ph}^{min}$  (or minimum optical power) of the photonic link should be determined using Eq. (46) or Eq. (47). This is to make certain that the dynamic range of the system is preserved. Third, a modulator with low enough  $V_{\pi}$  should be chosen, and the RF loss of the photonic link should be evaluated by substituting  $V_{\pi}$  and  $I_{ph}^{min}$  in Eq. (1). Fourth, a preamplifier should be selected with a gain large enough to compensate for the link's RF loss. To ensure that the preamplifier does not limit the dynamic range of the system, its intercept point and 1-dB compression should satisfy Eqs. (7) and (8), respectively. Finally, the noise factor degradation of the system caused by the insertion of the photonic link should be evaluated using Eqs. (33) and (34).

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#### References

- G. E. Betts, L. M. Johnson, C. H. Cox III, and S. D. Lowney, "High-Performance Optical Analog Link Using External Modulator," *IEEE Photonics Technology Letters*, vol. 1, no. 11, pp. 404-406, November 1989.
- [2] R. T. Logan and G. F. Lutes, "High Stability Microwave Fiber Optic Systems: Demonstration and Applications," *Proceedings of the 46th Annual Symposium on Frequency Control*, Hershey, Pennsylvania, pp. 310-316, May 27-29, 1992.
- [3] W. E. Stephens and T. R. Joseph, "System Characteristics of Direct Modulated and Externally Modulated RF Fiber-Optic Links," *Journal of Lightwave Technology*, vol. LT-5, no. 3, pp. 380-387, March 1987.
- [4] W. L. Glomb, "Fiber Optic Links for Antenna Remoting," SPIE Proceedings, vol. 1703, pp. 523-527, 1992.
- [5] W. L. Glomb, J. D. Farina, and S. W. Merritt, "Minimum Noise Figure Microwave Transmission System," SPIE Proceedings, vol. 1703, pp. 528-534, 1992.
- [6] R. T. Logan, G. F. Lutes, and L. Maleki, "Microwave Analog Fiber Optic Link for Use in the NASA/JPL Deep Space Network," The Telecommunications and Data Acquisition Progress Report 42-100, vol. October-December 1989, Jet Propulsion Laboratory, Pasadena, California, pp. 21-33, February 15, 1990.

- [7] B. H. Kolner and D. W. Dolfi, "Intermodulation Distortion and Compression in an Integrated Electrooptic Modulator," *Applied Optics*, vol. 26, no. 17, pp. 3676–3680, September 1987.
- [8] D. E. Norton, "The Cascading of High Dynamic Range Amplifiers," *Microwave Journal*, vol. 16, no. 6, pp. 57-71, June 1973.
- [9] T. J. Kane, "Intensity Noise in Diode-Pumped Single-Frequency Nd:YAG Lasers and Its Control by Electronic Feedback," *IEEE Photonics Technology Letters*, vol. 2, no. 4, pp. 244-245, April 1990.
- [10] D. M. Dagenais, K. P. Koo, and D. Dandridge, "Demonstration of Low-Frequency Intensity Noise Reduction for Fiber Sensors Powered by Diode-Pumped Nd-YAG Lasers," *IEEE Photonics Technology Letters*, vol. 4, no. 5, pp. 518-520, May 1992.

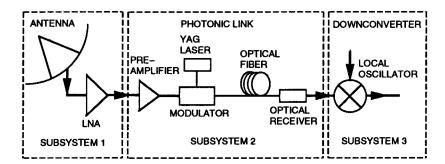
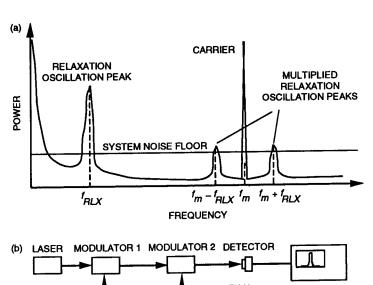


Fig. 1. An illustration of a photonic link in an analog system and how to group the system into three subsystems.



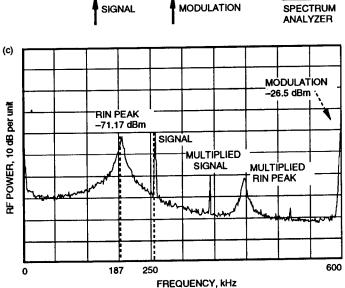


Fig. 2. The influence of multiplied relaxation oscillation peaks on the system: (a) the laser relaxation oscillation peak is multiplied up by the modulation signal. The multiplied peaks may be mistaken for the signal when they are above the system noise floor; (b) diagram of an experiment to verify that the strength of the multiplied noise peak obeys the same rule as a multiplied signal; and (c) results of the experiment.

### Appendix A

# Derivation of Gain, 1-dB Compression, and Intercept Point of an Optical Link

The optical transmission function of a Mach-Zehnder modulator biased at 50 percent of its transmission peak is

$$T(t) = \frac{1}{2} \left[ 1 - \sin \frac{\pi V(t)}{V_{\tau}} \right]$$
 (A-1)

where  $V_{\pi}$  is the half-wave voltage of the modulator and V(t) is the voltage of the driving signal. For  $|\pi V(t)/V_{\pi}| < 1$ , we can expand Eq. (A-1) into a Taylor series:

$$T(t) \approx \frac{1}{2} \left\{ 1 - \frac{\pi V(t)}{V_{\pi}} + \frac{1}{6} \left[ \frac{\pi V(t)}{V_{\pi}} \right]^{3} \right\}$$
 (A-2)

The error of the approximation is less than 0.83 percent.

#### A. Gain of the Optical Link

For a single-tone driving signal  $V(t) = V_o \sin \omega t$ , Eq. (A-2) becomes

$$T(t) = \frac{1}{2} \left[ 1 - m \left( 1 - \frac{m^2}{8} \right) \sin \omega t - \frac{m^3}{24} \sin 3\omega t \right]$$
 (A-3)

where m is the modulation depth and is defined as

$$m \equiv \frac{\pi V_o}{V_{\pi}} \tag{A-4}$$

The total photocurrent current I(t) in the load resistor of the photoreceiver is

$$I(t) = \alpha \eta W_{in} T(t) = I_{ph} T(t) \tag{A-5}$$

where  $I_{ph} = \alpha \eta W_{in}$  is the total average photocurrent in the load resistor,  $W_{in}$  is the input optical power to the modulator,  $\eta$  is the responsivity of the photoreceiver, and  $\alpha$  is the total optical loss of the optical link, including modulator insertion loss, fiber attenuation loss, and optical coupling loss.

In the frequency domain, the optical link's output power  $P_{op}(\omega)$  of the fundamental frequency component is

$$P_{op}(\omega) = I^2(\omega) \frac{R_L}{2} = \frac{1}{2} I_{ph}^2 R_L m^2 \left( 1 - \frac{m^2}{8} \right)^2$$
 (A-6)

On the other hand, the total input RF power  $P_m(\omega)$  to the modulator is

$$P_m(\omega) = \frac{V_o^2}{2R_m} = \frac{m^2 V_\pi^2}{2\pi^2 R_m}$$
 (A-7)

where  $R_m$  is the input impedance of the modulator. Eq. (A-4) is used in deriving Eq. (A-7).

Substituting Eq. (A-7) in Eq. (A-6), we obtain

$$P_{op} = G_{op} \left( 1 - \frac{G_{op} P_m}{4I_{ph}^2 R_L} \right)^2 P_m \tag{A-8}$$

where  $G_{op}$  is the small signal gain of the optical link and is defined as

$$G_{op} \equiv \frac{P_{op}}{P_m} = \pi^2 \frac{I_{ph}^2 R_L}{V_\pi^2 / R_m}$$
 (A-9)

#### B. The 1-dB Compression of the Optical Link

From Eq. (A-8), 1-dB compression occurs when  $(1 - G_{op}P_m/4I_{ph}^2R_L)^2 = 0.8$ . Therefore, the input 1-dB compression of the optical link is

$$P_m^{\text{1dB}} \approx \frac{2I_{ph}^2 R_L}{5G_{op}} \tag{A-10}$$

#### C. Third-Order Intercept of the Optical Link

For a two-tone driving signal of equal amplitude  $V(t) = V_o(\sin \omega_1 t + \sin \omega_2 t)$ , Eq. (A-2) becomes

$$T(t) pprox rac{1}{2} \left\{ \left[ 1 - m \left( 1 - rac{3m^2}{8} 
ight) \left( \sin \omega_1 t + \sin \omega_2 t 
ight) 
ight]$$

$$+\frac{m^3}{8}\left[\sin\left(\omega_1-\Delta\omega\right)+\sin\left(\omega_2+\Delta\omega\right)\right]$$
 (A-11)

where  $\Delta \omega = \omega_2 - \omega_1$ . In Eq. (A-11), the second term is the intermodulation product and other higher harmonic terms were neglected. Similar to the derivation of Eq. (A-6), the RF power of the output intermodulation product is

$$P_{IM}(\omega_1 - \Delta\omega) = P_{IM}(\omega_2 + \Delta\omega) = \frac{I_{ph}^2 R_L m^6}{128} \quad (A-12)$$

Substitution of Eqs. (A-7) and (A-9) in Eq. (A-12) yields the output intermodulation products

$$P_{IM} = \frac{(G_{op} P_m)^3}{\left(4I_{ph}^2 R_L\right)^2} \tag{A-13}$$

At the third-order intercept point,  $P_{IM} = G_{op}P_m$ . Substituting in Eq. (A-13), we obtain

$$IP_m = P_m = \frac{4I_{ph}^2 R_L}{G_{op}}$$
 (A-14)

# Appendix B

# **Multiplied RIN Noise**

The total optical power incident on the photoreceiver is

$$\Delta P(t) = [P_o + \Delta P(t)] (1 + m \sin \omega_m t)$$
 (B-1)

where  $\Delta P(t)$  is the optical power fluctuation,  $\omega_m$  is the modulation frequency, and m is the modulation depth defined in Eq. (A-4). Because the optical fluctuation of a laser peaks at relaxation oscillation frequency  $\omega_R$ , as an approximation,  $\Delta P(t)$  can be written as

$$\Delta P(t) = \Delta P_o(t) \sin \omega_R t$$
 (B-2)

Substituting Eq. (B-2) in Eq. (B-1), we obtain

$$P(t) = P_o (1 + m \sin \omega_m t) + \Delta P_o(t) \sin \omega_R t$$

$$-\frac{m\Delta P_o(t)}{2}\left[\cos\left(\omega_m - \omega_R\right) - \cos\left(\omega_m + \omega_R\right)t\right]$$
(B-3)

The photocurrent at  $\omega_m - \omega_R$  and  $\omega_m + \omega_R$  is

$$I_{mRIN} = \frac{\eta \Delta P_o(t)m}{2}$$
 (B-4)

The corresponding rms noise power at  $\omega_m - \omega_R$  and  $\omega_m + \omega_R$  is

$$P_{mRIN} = \frac{I_{mRIN}^2 R_L}{2} = \frac{1}{2} \left(\frac{\eta m}{2}\right)^2 R_L < \Delta P_0(t)^2 >$$
(B-5)

where <> denotes for time average.

Because  $RIN = \langle \Delta P_0(t)^2 \rangle / P_o^2$ , the last equation becomes

$$P_{mRIN} = \frac{1}{2} \left( \frac{I_{ph} m}{2} \right)^2 R_L RIN \tag{B-6}$$

Substitution of Eqs. (A-7) and (A-9) in Eq. (B-5) yields

$$P_{mRIN} = \frac{1}{4} G_{op} P_m RIN \tag{B-7}$$